

# Tunable magnetization relaxation in spin valves

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In spin valves the damping parameters of the free layer are determined non-locally by the entire magnetic configuration. In a dual spin valve structure that comprises a free layer embedded between two pinned layers, the spin pumping mechanism, in combination with the angular momentum conservation, renders the tensor-like damping parameters tunable by varying the interfacial and diffusive properties. Simulations based on the Landau-Lifshitz-Gilbert phenomenology for a macrospin model are performed with the tensor-like damping and the relaxation time of the free layer magnetization is found to be largely dependent on while tunable through the magnetic configuration of the source-drain magnetization.

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A thorough knowledge of magnetization relaxation holds the key to understand magnetization dynamics in response to applied fields<sup>1</sup> and spin-transfer torques.<sup>2,3</sup> In the framework of Landau-Lifshitz-Gilbert (LLG) phenomenology, relaxation is well captured by the Gilbert damping parameter that is usually cited as a scalar quantity. As pointed out by Brown half a century ago,<sup>4</sup> the Gilbert damping for a single domain magnetic particle is in general a tensor.

When a ferromagnetic thin film is deposited on a normal metal substrate, an enhanced damping has been observed ferromagnetic resonance experiments.<sup>5</sup> This observation is successfully explained by spin pumping:<sup>6,7</sup> The slow precession of the magnetization pumps spin current into the adjacent normal metal where the dissipation of spin current provides a non-local mechanism to the damping. The damping enhancement is found to be proportional to spin mixing conductance, a quantity playing key roles in the magneto-electronic circuit theory.<sup>7,8</sup>

The pumped spin current  $I_p \propto \mathbf{M} \times \dot{\mathbf{M}}$  is always in the plane formed by the free layer magnetization direction  $\mathbf{M}$  and the instantaneous axis about which the magnetization precesses. Therefore, in a single spin valve, when  $\mathbf{M}$  is precessing around the source (drain) magnetization  $\mathbf{m}$ , the pumping current is always in the plane of  $\mathbf{m}$  and  $\mathbf{M}$ .<sup>9</sup> Let us assume an azimuth angle  $\theta$  between  $\mathbf{m}$  and  $\mathbf{M}$ . In such an *in-plane* configuration, the pumping current  $I_p$  has a component  $I_p \sin \theta$  that is parallel to  $\mathbf{m}$ . The spin transfer torque acting on the source (drain) ferromagnet  $\mathbf{m}$  is the component of spin current that is in the plane and perpendicular to  $\mathbf{m}$ . To simplify the discussion, we consider it to be completely absorbed by  $\mathbf{m}$ . The longitudinal (to  $\mathbf{m}$ ) component experiences multiple reflection at the source (drain) contact, and cancels the damping torque by an amount proportional to  $I_p \sin^2 \theta$  but is still aligned along the direction of  $\mathbf{M} \times \dot{\mathbf{M}}$ . Therefore the total damping parameter has an angle  $\theta$  dependence but still picks up a scalar (isotropic) form. This is the well-known dynamic stiffness explained by Tserkovnyak *et al.*<sup>9</sup> In the most general case, when the precessing axis of the free layer is mis-aligned with  $\mathbf{m}$ , there is always an

*out-of-plane* pumping torque perpendicular to the plane. In the paradigm of Slonczewski, this *out-of-plane* component is not absorbed at the interface of the source (drain) ferromagnetic nodes, while the conservation of angular momentum manifests it as a damping enhancement that shows the tensor form when installed in the LLG equation.

Studies in lateral spin-flip transistors have suggested a tensor form for the enhanced damping parameters.<sup>10</sup> In spin valves, works based on general scattering theory have discussed the damping in the framework of fluctuation-dissipation theorem<sup>11</sup> and shown that the Gilbert damping tensor can be expressed using scattering matrices,<sup>12</sup> thus enabling first-principle investigation.<sup>13</sup> But explicit analytical expressions of the damping tensor, its dependence on the magnetic configuration as well as the material properties and particularly its impact on the magnetization relaxation are largely missing.

In this paper, we investigate the Gilbert damping parameters of the free layer in the so-called dual spin valve (DSV).<sup>14–16</sup> We analyze the origin of the damping tensor and derive explicit analytical expressions of its non-local dependence on the magnetic configuration and materials properties. A generalization of our damping tensor to a continuous magnetic texture agrees well with the results in earlier works. Particularly, we show, in numeric simulations, that by tuning the magnetic configurations of the entire DSV, the relaxation time of the free layer can be increased or decreased.

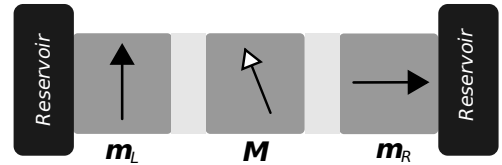


FIG. 1: A dual spin valve consists of a free layer (with magnetization direction  $\mathbf{M}$ ) sandwiched by two fixed ferromagnetic layers (with magnetization directions  $\mathbf{m}_L$  and  $\mathbf{m}_R$ ) through two normal metal spacers. The fixed layer are attached to reservoirs.

To analyze the spin and charge currents in a DSV, we employ the magneto-electronic circuit theory and spin pumping,<sup>7,8</sup> in combination with diffusion equations.<sup>17</sup> Pillar-shaped metallic spin valves usually consist of normal-metal ( $N$ ) spacers much shorter than its spin-flip relaxation length, see for example Ref.[3,15]. To a good approximation, in the  $N$  nodes, a spatially homogeneous spin accumulation is justified and the spin current ( $\mathbf{I}_i$ ) conservation dictates  $\sum_i \mathbf{I}_i = 0$  (where subscript  $i$  indicates the source of spin current).

A charge chemical potential ( $\mu$ ) and a spin accumulation ( $\mathbf{s}$ ) are assigned to every  $F$  or  $N$  node. In a transition metal ferromagnet, a strong exchange field aligns the spin accumulation to the magnetization direction. At every  $F|N$  interface, the charge and spin currents on the  $N$  side are determined by the contact conductance and the charge and spin distributions on both sides of the contact. For example, at the contact between the left lead ferromagnet to the left normal metal  $N_1$ , called  $L|N_1$  thereafter, the currents are<sup>8</sup>

$$\begin{aligned} I_L &= \frac{e}{2h} G_L [(\mu_1 - \mu_L) + P_L(\mathbf{s}_1 - \mathbf{s}_L) \cdot \mathbf{m}_L], \\ \mathbf{I}_L &= -\frac{G_L}{8\pi} [2P_L(\mu_1 - \mu_L)\mathbf{m}_L + (\mathbf{s}_1 - \mathbf{s}_L) \cdot \mathbf{m}_L \mathbf{m}_L \\ &\quad + \eta_L(\mathbf{s}_1 - \mathbf{s}_1 \cdot \mathbf{m}_L \mathbf{m}_L)]. \end{aligned} \quad (1)$$

We have used the notation  $G = g^\uparrow + g^\downarrow$  is the sum of the spin- $\sigma$  interface conductance  $g^\sigma$ . The contact polarisation  $P = (g^\uparrow - g^\downarrow)/(g^\uparrow + g^\downarrow)$ . The ratio  $\eta = 2g^{\uparrow\downarrow}/G$  is between the real part of the spin-mixing conductance  $g^{\uparrow\downarrow}$  and the total conductance  $G$ . The imaginary part of  $g^{\uparrow\downarrow}$  is usually much smaller than its real part, thus discarded.<sup>18</sup> The spin-coherence length in a transition metal ferromagnet is usually much shorter than the thickness of the thin film,<sup>19</sup> which renders the mixing transmission negligible.<sup>7</sup> The precession of the free layer magnetization  $\mathbf{M}$  pumps a spin current  $\mathbf{I}_p = (\hbar/4\pi)g_F^{\uparrow\downarrow}\mathbf{M} \times \dot{\mathbf{M}}$  into the adjacent normal nodes  $N_1$  and  $N_2$ , which is given by the mixing conductance  $g_F^{\uparrow\downarrow}$  at the  $F|N_{1(2)}$  interface (normal metals spacers are considered identical on both sides of the free layer).

A back flow spin current at the  $F|N_1$  interface reads

$$\begin{aligned} \mathbf{I}_1 &= -\frac{G_F}{8\pi} [2P_F(\mu_1 - \mu_F)\mathbf{M} + (\mathbf{s}_1 - \mathbf{s}_F) \cdot \mathbf{M} \mathbf{M} \\ &\quad + \eta_F(\mathbf{s}_1 - \mathbf{s}_1 \cdot \mathbf{M} \mathbf{M})] \end{aligned} \quad (2)$$

on the  $N_1$  side. Therefore, a weak spin-flip scattering in  $N_1$  demands  $\mathbf{I}_L + \mathbf{I}_1 + \mathbf{I}_p = 0$ , which is dictated by angular momentum conservation. The same conservation law rules in  $N_2$ , where  $\mathbf{I}_R + \mathbf{I}_2 + \mathbf{I}_p = 0$ .

For the ferromagnetic ( $F$ ) nodes made of transition metals, the spin diffusion is taken into account properly.<sup>9</sup> In a strong ferromagnet, any transverse components decay quickly due to the large exchange field, thus the longitudinal spin accumulation  $\mathbf{s}_\nu = s_\nu \mathbf{m}_\nu$  (with  $\nu = L, R, F$ ) diffuses and decays exponentially at a length scale given by spin diffusion length ( $\lambda_{sd}$ ) as  $\nabla_x^2 s_\nu =$

$s_\nu/\lambda_{sd}$ . The difference in spin-dependent conductivity of majority and minority carriers is taken into account by enforcing the continuity of longitudinal spin current  $\mathbf{m}_\nu \cdot \mathbf{I}_\nu = -(D_\nu^\uparrow \nabla_x s_\nu^\uparrow - D_\nu^\downarrow \nabla_x s_\nu^\downarrow)$  at the every  $F|N$  interface. We assume vanishing spin currents at the outer interfaces to reservoirs.

The diffusion equations and current conservation determine, self-consistently, the spin accumulations and spin currents in both  $N$  and  $F$  nodes. We are mainly concerned with the exchange torque<sup>9</sup>  $\mathbf{T} = -\mathbf{M} \times (\mathbf{I}_L + \mathbf{I}_R) \times \mathbf{M}$  acting on  $\mathbf{M}$ . A general analytical formula is attainable but lengthy. In the following, we focus on two scenarios that are mostly relevant to the state-of-the-art experiments in spin valves and spin pumping: (1) The free layer has a strong spin flip (short  $\lambda_{sd}$ ) and the thickness  $d_F \geq \lambda_{sd}$ , for which the permalloy (Py) film is an ideal candidate;<sup>15</sup> (2) The free layer is a half metal, such as  $\text{Co}_2\text{MnSi}$  studied in a recent experiment.<sup>20</sup>

*Strong spin flip in free layer.* We assume a strong spin flip scattering in the free layer i.e.,  $d_F \geq \lambda_{sd}$ . We leave the diffusivity properties in the lead  $F$  nodes arbitrary. The total exchange torque is partitioned into two parts: An *isotropic* part that is parallel to the direction of the Gilbert damping  $\mathbf{M} \times \dot{\mathbf{M}}$  and an *anisotropic* part that is perpendicular to the plane spanned by  $\mathbf{m}_{L(R)}$  and  $\mathbf{M}$  (or the projection of  $\mathbf{M} \times \dot{\mathbf{M}}$  to the direction  $\mathbf{m}_{L(R)} \times \mathbf{M}$ ), i.e.,

$$\begin{aligned} \mathbf{T} &= \hbar \frac{g_F^{\uparrow\downarrow}}{4\pi} (\mathfrak{D}_{is}^L + \mathfrak{D}_{is}^R) (\mathbf{M} \times \dot{\mathbf{M}}) \\ &\quad + \hbar \frac{g_F^{\uparrow\downarrow}}{4\pi} \mathbf{M} \times [(\mathfrak{D}_{an}^L \hat{A}_{L,an} + \mathfrak{D}_{an}^R \hat{A}_{R,an}) \dot{\mathbf{M}}], \end{aligned} \quad (3)$$

where the material-dependent parameters  $\mathfrak{D}_{is}^{L(R)}$  and  $\mathfrak{D}_{an}^{L(R)}$  are detailed in the Appendix A.

Most interest is in the anisotropic damping described by a symmetric tensor with elements

$$\hat{A}_{an}^{ij} = -m_i m_j \quad (4)$$

where  $i, j = x, y, z$  (we have omitted the lead index  $L$  or  $R$ ). The elements of  $\hat{A}_{an}$  are given in Cartesian coordinates of the source-drain magnetization direction. The anisotropic damping appears as  $\mathbf{M} \times \hat{A}_{an} \dot{\mathbf{M}}$  that is always perpendicular to the free layer magnetization direction, thus keeping the length of  $\mathbf{M}$  constant.<sup>11</sup> It is not difficult to show that when  $\mathbf{M}$  is precessing around  $\mathbf{m}$ , the anisotropic part vanishes due to  $\hat{A}_{an} \dot{\mathbf{M}} = 0$ .

We generalize Eq.(4) to a continuous magnetic texture. Consider here only one-dimensional spatial dependence and the extension to higher dimensions is straightforward. The Cartesian component of vector  $\mathbf{U} \equiv \mathbf{M} \times \hat{A}_{an} \dot{\mathbf{M}}$  is  $U_i = -\varepsilon_{ijk} M_j m_k m_l \dot{M}_l$  (where  $\varepsilon_{ijk}$  is the Levi-Civita tensor and repeated indices are summed). We assume the fixed layer and the free layer differ in space by a lattice constant  $a_0$ , which allows  $m_k \approx M_k(x + a_0)$ . A Taylor expansion in space leads to  $\mathbf{U} = -a_0^2 \mathbf{M} \times (\hat{\partial} \dot{\mathbf{M}})$ , where the matrix elements  $\hat{\partial}_{kl} = (\partial_x \mathbf{M})_k (\partial_x \mathbf{M})_l$  and we

have assumed that the magnetization direction is always perpendicular to  $\partial_x \mathbf{M}$ . In this case, three vectors  $\partial_x \mathbf{M}$ ,  $\mathbf{M} \times \partial_x \mathbf{M}$  and  $\mathbf{M}$  are perpendicular to each other. A rotation around  $\mathbf{M}$  by  $\pi/2$  leaves  $\mathbf{M}$  and  $\dot{\mathbf{M}}$  unchanged while interchanging  $\partial_x \mathbf{M}$  with  $\mathbf{M} \times \partial_x \mathbf{M}$ , we have

$$\hat{\mathcal{D}}_{kl} = (\mathbf{M} \times \partial_x \mathbf{M})_k (\mathbf{M} \times \partial_x \mathbf{M})_l, \quad (5)$$

which agrees with the so-called differential damping tensor Eq.(11) in Ref.[21].

Eq.(3) suggests that the total exchange torque on the free layer is a linear combination of two independent exchange torques arising from coupling to the left and the right  $F$  nodes. This form arises due to a strong spin-flip scattering in the free layer that suppresses the exchange between two spin accumulations  $\mathbf{s}_1$  and  $\mathbf{s}_2$  in the  $N$  nodes. In the pursuit of a concise notation for the Gilbert form, the exchange torque can be expressed as  $\mathbf{T} = \mathbf{M} \times \overleftrightarrow{\alpha} \dot{\mathbf{M}}$  with a total damping tensor given by

$$\overleftrightarrow{\alpha} = \hbar \frac{g_F^{\uparrow\downarrow}}{4\pi} \left( \mathcal{D}_{is}^L + \mathcal{D}_{is}^R + \mathcal{D}_{an}^L \hat{A}_{L,an} + \mathcal{D}_{an}^R \hat{A}_{R,an} \right). \quad (6)$$

The damping tensor  $\overleftrightarrow{\alpha}$  is determined by the entire magnetic configuration of the DSV and particularly by the conductance of  $F|N$  contacts and the diffusive properties the  $F$  nodes.

*Half metallic free layer.* This special while experimentally relevant<sup>20</sup> case means  $P_F = 1$ . Half-metallicity in combination with the charge conservation enforces a longitudinal back flow that is determined solely by the bias current: The spin accumulations in  $N$  nodes do not contribute to the spin accumulation inside the free layer, thus an independent contribution due to left and right leads is foreseen. We summarize the material specific parameters in the Appendix A. When spin flip is weak in the source-drain ferromagnets,  $\xi_L \approx 0$  leads to  $\mathcal{D}_{is} \approx 0$ . In this configuration, by taking a (parallel or anti-parallel) source-drain magnetization direction as the precessing axis, the total damping enhancement vanishes, which reduces to the scenario of  $\nu = 1$  in Ref.[9].

*Magnetization relaxation.* To appreciate the impact of an anisotropic damping tensor on the magnetization relaxation, we perform a simulation, for the free layer magnetization, using Landau-Lifshitz-Gilbert (LLG) equation augmented by the tensor damping, i.e.,

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \alpha_0 \mathbf{M} \times \frac{d\mathbf{M}}{dt} + \frac{\gamma}{\mu_0 M_s V} \mathbf{M} \times \overleftrightarrow{\alpha} \frac{d\mathbf{M}}{dt}. \quad (7)$$

$\alpha_0$  is the (dimensionless) intrinsic Gilbert damping parameter. Symbol  $\gamma$  is the gyromagnetic ratio,  $M_s$  is the saturation magnetization, and  $V$  is the volume of the free layer.  $\mu_0$  stands for the vacuum permeability. The dynamics under the bias-driven spin transfer torque is not the topic in this paper, but can be included in a straightforward way.<sup>22</sup> We give in the Appendix B the expressions of the bias-driven spin torques.

We are mostly interested in the relaxation of the magnetization, instead of particular magnetization trajectories, in the presence of a tensor damping. The following simulation is performed for the scenario where the free layer has a strong spin flip, i.e., Case (1). We employ the pillar structure from Ref.[15] while considering the free layer (Py) to be 8nm thick (a thicker free layer favors a better thermal stability.<sup>15</sup>) The source-drain ferromagnets are cobalt (Co) and we expect the results are valid for a larger range of materials selections. The Py film is elliptic with three axes given by  $2a = 90$  nm,  $2b = 35$  nm,<sup>15</sup> and  $c = 8$  nm. The demagnetizing factors  $\mathcal{D}_{x,y,z}$  in the shape anisotropy energy  $E_{dem} = (1/2)\mu_0 M_s^2 V \sum_{i=x,y,z} \mathcal{D}_i M_i^2$  are  $\mathcal{D}_x = 0.50$ ,  $\mathcal{D}_y = 0.37$  and  $\mathcal{D}_z = 0.13$ . An external field  $\mathbf{H}_a$  leads to a Zeeman splitting  $E_{Zee} = -V\mu_0 M_s \mathbf{H}_a \cdot \mathbf{M}$ . For Py films, we neglect the uniaxial anisotropy. The total free energy  $E_T = E_{Zee} + E_{dem}$  gives rise to an effective field  $\mathbf{H}_{eff} = -(1/V M_s \mu_0) \partial E_T / \partial \mathbf{M}$ .

The spin-dependent conductivities in the bulk of Co and the spin diffusion length  $\lambda_{Co} \approx 60$  nm are taken from the experimental data.<sup>24</sup> For Py, we take  $\lambda_{Py} \approx 4$  nm.<sup>25</sup> To have direct connection with experiments, the above mentioned bare conductance has to be renormalized by the Sharvin conductance.<sup>26</sup> For Py/Cu the mixing conductance, we take the value  $g_F^{\uparrow\downarrow} S^{-1} \approx 15$  nm<sup>-2</sup>,<sup>26</sup> which gives  $\Re_{L(R)F} \approx 1.0$ .

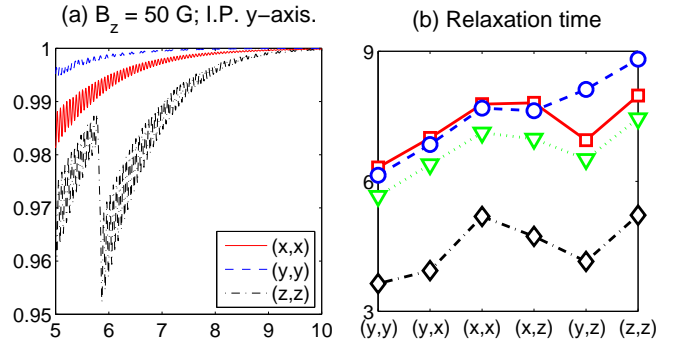


FIG. 2: (Color online)  $M_z$  as a function of time (in ns) in presence of different source-drain magnetic configurations and applied fields. (a) The external magnetic field  $B_z = 50$  Gauss is applied along  $z$ -axis. The blue (dashed), red (solid) and black (dotted dash) curves correspond to source-drain magnetization in configurations  $(y, y)$ ,  $(x, x)$ , and  $(z, z)$  respectively. (b) Magnetization relaxation times (in the unit of ns) versus source-drain magnetic configurations at different applied field along  $z$ -axis.:  $B_z = 10$  G (red  $\square$ ),  $B_z = 50$  G (blue  $\circ$ ),  $B_z = 200$  G (green  $\nabla$ ),  $B_z = 800$  G (black  $\diamond$ ). Lines are a guide for the eyes. The initial position (I.P.) of the free layer is taken along  $y$ -axis.

The relaxation time  $\tau_r$  is extracted from the simulations by demanding at a specific moment  $\tau_r$  the  $|M_z - 1.0| < 10^{-3}$ , i.e., reaches the easy axis. In the absence of bias, panel (a) of Fig.2 shows the late stage of magnetization relaxation from an initial position ( $y$ -

axis) in the presence of an tensor damping, under various source-drain (SD) magnetic configurations. The results are striking: Under the same field, switching the SD configurations increases or decreases  $\tau_r$ . In panel (b), the extracted relaxation times  $\tau_r$  versus SD configurations under various fields are shown. At low field  $B_z = 10$  G (red  $\square$ ), when switching from  $(z, z)$  to  $(y, y)$ ,  $\tau_r$  is improved from 8.0 ns to 6.3 ns, about 21%. At a higher field  $B_z = 800$  G (black  $\diamond$ ), the improvement is larger from 5.2 at  $(z, z)$  to 3.6 at  $(y, y)$ , nearly 31%. To a large trend, the relaxation time improvement is more significant at higher applied fields.

In conclusion, combining conservation laws and magneto-electronic circuit theory, we have analyzed the Gilbert damping tensor of the free layer in a dual spin valve. Analytical results of the damping tensor as functions of the entire magnetic configuration and material properties are obtained. Numerical simulations on LLG equation augmented by the tensor damping reveal a tunable magnetization relaxation time by a strategic selection of source-drain magnetization configurations. Results presented in this paper open a new venue to the design and control of magnetization dynamics in spintronic applications.

X.Wang is indebted to G. E. W. Bauer, who has brought the problem to his attention and offered invaluable comments.

### Appendix A: Material dependent parameters

In this paper,  $\Re_{L(R)F} \equiv g_{L(R)}^{\uparrow\downarrow}/g_F^{\uparrow\downarrow}$  is the mixing conductance ratio and  $\chi_{L(R)} \equiv \mathbf{m}_{L(R)} \cdot \mathbf{M}$ . The diffusivity parameter  $\xi_{L(R)} = \phi_{L(R)}(1 - P_{L(R)}^2)/\eta_{L(R)}$ , where for the left  $F$  node

$$\phi_L = \frac{1}{1 + \frac{(\sigma_L^{\uparrow} + \sigma_L^{\downarrow})\lambda_L e^2}{4hS\sigma_L^{\uparrow}\sigma_L^{\downarrow} \tanh(d_L/\lambda_L)} G_L(1 - P_L^2)} \quad (\text{A1})$$

where  $h$  the Planck constant,  $S$  the area of the thin film,  $e$  the elementary charge,  $\lambda_L$  the spin diffusion length,  $d_L$  the thickness of the film, and  $\sigma^{\uparrow(\downarrow)}$  the spin-dependent conductivity.  $\phi_R$  is obtained by substituting all  $L$  by  $R$  in Eq.(A1). Parameter  $\xi_F$  is given by  $\xi_F = (1 - P_F^2)\phi_F/\eta_F$  with

$$\phi_F = \frac{1}{1 + \frac{(\sigma_F^{\uparrow} + \sigma_F^{\downarrow})\lambda_F e^2}{4hS\sigma_F^{\uparrow}\sigma_F^{\downarrow}} G_F(1 - P_F^2)}. \quad (\text{A2})$$

The material dependent parameters as appearing in the damping tensor Eq.(6) are: (1) In the case of a strong

spin flip in free layer,

$$\begin{aligned} \mathfrak{D}_{is}^{L(R)} &= \frac{\Re_{L(R)F}}{\mathcal{L}_{L(R)F}} \left[ \xi_{L(R)} \Re_{L(R)F} + \xi_{L(R)} \xi_F (1 - \chi_{L(R)}^2) \right. \\ &\quad \left. + \xi_F (1 - \chi_{L(R)}^2) \chi_{L(R)}^2 \right], \\ \mathfrak{D}_{an}^{L(R)} &= \frac{\Re_{L(R)F}}{\mathcal{L}_{L(R)F}} \frac{(\xi_{L(R)} - 1)[\xi_F(1 - \chi_{L(R)}^2) + \Re_{L(R)F}]}{1 + \Re_{L(R)F}}, \\ \mathcal{L}_{L(R)F} &= (1 + \Re_{L(R)F} \chi_L^2) \xi_F (1 - \chi_{L(R)}^2) \\ &\quad + \Re_{L(R)F} \left[ (1 - \chi_{L(R)}^2)(1 + \xi_{L(R)} \xi_F) \right. \\ &\quad \left. + \xi_{L(R)} \Re_{L(R)F} + \xi_F \chi_{L(R)}^2 \right]; \end{aligned} \quad (\text{A3})$$

(2) In the case of a half metallic free layer

$$\begin{aligned} \mathfrak{D}_{is}^{L(R)} &= \frac{\Re_{L(R)F} \xi_{L(R)}}{(1 - \chi_{L(R)}^2) + \xi_{L(R)}(\chi_L^2 + \Re_{L(R)F})}, \\ \mathfrak{D}_{an}^{L(R)} &= \frac{\Re_{L(R)F}}{1 + \Re_{L(R)F}} \\ &\quad \times \frac{\xi_{L(R)} - 1}{(1 - \chi_{L(R)}^2) + \xi_{L(R)}(\chi_L^2 + \Re_{L(R)F})}. \end{aligned} \quad (\text{A4})$$

### Appendix B: Bias dependent spin torques

The full analytical expression of bias dependent spin torques are rather lengthy. We give here the expressions, under a bias current  $I$ , for symmetric SD ferromagnets (i.e.,  $\phi_L = \phi_R = \phi$  thus  $\xi_L = \xi_R = \xi$ ) with parallel or anti-parallel magnetization direction. (1) With a strong spin flip in the free layer, the parallel SD magnetization leads to vanishing bias-driven torque  $\mathbf{T}_{\uparrow\uparrow}^{(b)} = 0$ ; When the SD magnetizations are anti-parallelly (i.e.,  $\mathbf{m}_L = -\mathbf{m}_R \equiv \mathbf{m}$ ),

$$\begin{aligned} \mathbf{T}_{\uparrow\downarrow}^{(b)} &= \frac{IhP\phi}{e(1 + \Re)\mathcal{L}} [(\xi_F + \Re\xi_F\chi^2 + \Re)(1 - \chi^2) \\ &\quad + \Re(\Re + \xi_F(1 - \chi^2) + \chi^2)] \mathbf{m}_F \times (\mathbf{m} \times \mathbf{m}_F). \end{aligned} \quad (\text{B1})$$

(2) When the free layer is half metallic, for symmetric SD ferromagnets,  $\mathbf{T}_{\uparrow\uparrow}^{(b)} = 0$  and

$$\mathbf{T}_{\uparrow\downarrow}^{(b)} = \frac{Ih}{e} \frac{\phi P}{(1 - \xi)(1 - \chi^2) + \xi(\chi^2 + \Re)} \mathbf{m}_F \times (\mathbf{m} \times \mathbf{m}_F). \quad (\text{B2})$$

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